MULTIGLUON AMPLITUDES IN THE HIGH-ENERGY LIMIT[a]

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Abstract

We give a unified description of tree-level multigluon amplitudes in the high-energy limit. We represent the Parke-Taylor amplitudes and the Fadin-Kuraev-Lipatov amplitudes in terms of color configurations that are ordered in rapidity on a two-sided plot. We show that for the helicity configurations they have in common the Parke-Taylor amplitudes and the Fadin-Kuraev-Lipatov amplitudes coincide.

Amplitudes with multi-parton final states have several applications in high-energy hadronic interactions. They are used in computing:

multi-jet events, which are of phenomelogical interest because they appear as background to top-quark and electroweak-boson production and to eventual Higgs-boson production and new-physics signals;

scaling violations in DIS at small x_{bj} [1];

azimuthal-angle correlations in dijet production at large rapidity intervals [2];

dijet production with large rapidity gaps [3].

Multi-parton amplitudes are also of interest *per se* because they yield the radiative corrections to the total parton cross section, which in the high-energy limit of perturbative QCD is predicted to have a power-like growth in the parton center-of-mass energy $\sqrt{\hat{s}}$ [4].

Multi-parton amplitudes have been computed in the high-energy limit by Fadin, Kuraev and Lipatov (FKL) [4], who considered the tree-level production of n gluons in parton-parton scattering in the limit of a strong rapidity ordering of the produced partons, assuming their transverse momenta to be all of the same size, Q. This kinematic configuration is termed *multiregge kinematics*. The amplitudes are given by the exchange of a gluon ladder between the scattering partons (Fig.3). FKL made also an ansatz for the leading logarithmic contribution, in $\ln(\hat{s}/Q^2)$, of the loop corrections to the multi-parton amplitudes, to all orders in α_s. This changes the form of the propagators of the gluons exchanged in the $\hat{t}$ channel, but preserves the ladder structure of the amplitudes.

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On the other hand, *exact* tree-level amplitudes for the production of n gluons have been computed by Parke and Taylor (PT) [5] in a helicity basis, for specific helicity configurations of the incoming and outgoing gluons. In a helicity basis the color structure of the tree-level amplitudes may be decomposed as a sum over all the noncyclic permutations of the gluon color flows. In ref. [6] we represented the color flows of the squared PT amplitudes in terms of color lines in the fundamental representation of $SU(N_c)$. Permuting the color flows the color lines appear twisted, however every configuration may be untwisted introducing a two-sided plot [7]. We showed that restricting the squared PT amplitudes to the multiregge kinematics only the untwisted configurations with the gluons ordered in rapidity on the two-sided plot contribute. In ref. [6] we worked with the squared PT amplitudes at leading N_c, however due to the incoherence of the leading N_c term in the color sum of the squared PT amplitudes, the color flows we consider there are the same as the ones of the PT amplitudes themselves. Thus, also for the PT amplitudes, for which no approximation in N_c is made, the leading color configurations are the ones with the gluons ordered in rapidity on the two-sided plot. Considering then the sum over the leading color flows of the PT amplitudes in the multiregge kinematics, we have shown [8] that for the helicity configurations they have in common the PT amplitudes and the FKL amplitudes are equal. In giving here the outline of the proof, we follow ref.[8].

We consider the production of $n+2$ gluons of momentum p_i, with $i = 0, ..., n+1$ and $n \geq 0$, in the scattering between two gluons of momenta p_A and p_B, and we assume that the produced gluons satisfy the multiregge kinematics, i.e. we require that the gluons

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are strongly ordered in rapidity y and have comparable transverse momentum,

$$y_0 \gg y_1 \gg ... \gg y_{n+1}; \qquad |p_{i\perp}| \simeq |p_\perp|. \tag{1}$$

Tree-level multigluon amplitudes in a helicity basis are known *exactly* in the *maximally helicity-violating* configurations $(-, -, +, ..., +)$ [5],

$$M(-, -, +, ..., +) = 2^{2+n/2} g^{n+2} \sum_{[A,0,...,n+1,B]} \text{tr}(\lambda^a \lambda^{d_0} \cdots \lambda^{d_{n+1}} \lambda^b) \frac{\langle p_i p_j \rangle^4}{\langle \tilde{p}_A p_0 \rangle \cdots \langle p_{n+1} \tilde{p}_B \rangle \langle \tilde{p}_B \tilde{p}_A \rangle}, \tag{2}$$

where the $\pm$ signs on the left hand side label the gluon helicities, the λ's are the color matrices in the fundamental representation of $\text{SU}(N_c)$ and the sum is over the noncyclic permutations of the set $[A, 0, ..., B]$, i and j are the gluons of negative helicity, and we consider all the momenta as outgoing. The spinor products $\langle p_i p_j \rangle$ are defined in ref.[9].

In the multiregge kinematics the PT amplitudes (2) for which the numerator is the largest are the ones for which the pair of negative-helicity gluons is one of the following,

$$(A, B), \quad (A, n+1), \quad (B, 0), \quad (0, n+1). \tag{3}$$

We focus on the first pair, and fix $p_i = \tilde{p}_A = -p_A$ and $p_j = \tilde{p}_B = -p_B$ in eq.(2). The modifications needed for the other helicity configurations of eq.(3) may be found in ref.[8].

Then we examine all the color orderings, starting with the ordering $[A, 0, ..., n+1, B]$ (Fig.1). Computing the string of spinor products in the denominator of eq.(2) we find [8],

$$\langle p_A p_0 \rangle \cdots \langle p_{n+1} p_B \rangle \langle p_B p_A \rangle \simeq (-1)^{n+1} \langle p_A p_B \rangle^2 \prod_{i=0}^{n+1} p_{i\perp}. \tag{4}$$

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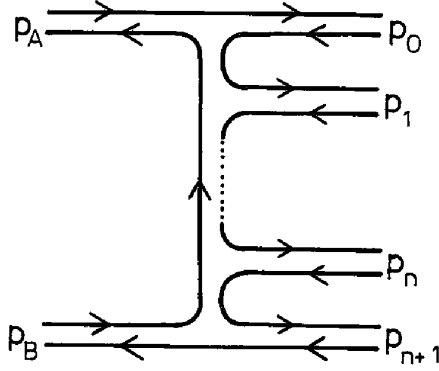
Figure 1: PT amplitude with color ordering $[A, 0, ..., n+1, B]$.

Every other color configuration, for which we keep fixed the position of gluons A and B in the color ordering and permute the outgoing gluons, gives a subleading contribution, of $\mathcal{O}(e^{-|y_i - y_j|})$, to eq.(2). We note that untwisting the color lines on a configuration with permuted outgoing gluons, the color ordering we obtain is different from the rapidity ordering. Thus the leading color configuration in multiregge kinematics is the one whose untwisted lines respect the rapidity ordering (Fig.1).

Next, we move gluon B one step to the left and consider the color orderings $[A, 0, ..., j-1, j+1, ..., n+1, B, j]$, with $j = 0, ..., n+1$ (Fig.2a). Untwisting the color lines, we get gluon j on the back of the plot (Fig.2b). We compute then the string of spinor products in eq.(2), and we note that the result is independent of which gluon we have taken to the back of the plot in Fig.2b. As before every permutation of the gluons on the front of the plot of Fig.2b gives a subleading contribution, of $\mathcal{O}(e^{-|y_i - y_j|})$, to eq.(2). Thus the leading color configurations are the $(n+2)$ configurations whose untwisted lines respect
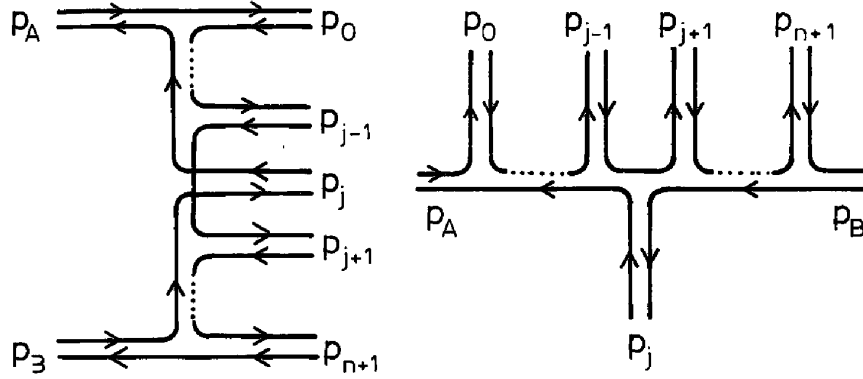
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Figure 2: *a)* PT amplitude with color ordering $[A, 0, ..., j-1, j+1, ..., n+1, B, j]$, and *b)* its untwisted version on the two-sided plot.

the rapidity ordering on the front of the plot of Fig.2b.

We can then proceed further by moving gluon B one more step to the left. Taking gluon B all the way to the left, we will have exhausted all the $(n+3)!$ noncyclic permutations of the color ordering $[A, 0, ..., B]$. Substituting then the leading contributions of the different color configurations into eq.(2), we obtain

$$M(-p_A, -; p_0, +; ...; p_{n+1}, +; -p_B, -) \simeq$$
$$(-1)^{n+1} \, 2^{2+n/2} \, g^{n+2} \, \hat{s} \, \frac{1}{\prod_{i=0}^{n+1} p_{i\perp}} \, \text{tr} \left(\lambda^a \, \left[\lambda^{d_0}, \left[\lambda^{d_1}, ..., \left[\lambda^{d_{n+1}}, \lambda^b \right] \right] \right] \right) , \qquad (5)$$

where the color orderings which contribute to eq.(5) in the multiregge kinematics are given by the 2^{n+2} configurations which respect the rapidity ordering on the two-sided plot.

The tree-level Fadin-Kuraev-Lipatov amplitude [4] for the production of $n+2$ gluons

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in the multiregge kinematics is given by,

$$
\begin{aligned}
M^{ad_0\ldots d_{n+1}b}_{\nu_A\nu_0\ldots\nu_{n+1}\nu_B} \quad \simeq \quad & 2\hat{s}\left(ig\,f^{ad_0c_1}\,\Gamma^{\mu_A\mu_0}\right)\,\epsilon^{\nu_A*}_{\mu_A}(p_A)\epsilon^{\nu_0}_{\mu_0}(p_0)\frac{1}{\hat{t}_1} \\
\cdot \quad & \left(ig\,f^{c_1d_1c_2}\,C^{\mu_1}(q_1,q_2)\right)\,\epsilon^{\nu_1}_{\mu_1}(p_1)\frac{1}{\hat{t}_2} \\
\cdot \quad & \\
\cdot \quad & \\
\cdot \quad & \left(ig\,f^{c_nd_nc_{n+1}}\,C^{\mu_n}(q_n,q_{n+1})\right)\,\epsilon^{\nu_n}_{\mu_n}(p_n)\frac{1}{\hat{t}_{n+1}} \\
\cdot \quad & \left(ig\,f^{bd_{n+1}c_{n+1}}\,\Gamma^{\mu_b\mu_{n+1}}\right)\,\epsilon^{\nu_B*}_{\mu_B}(p_B)\epsilon^{\nu_{n+1}}_{\mu_{n+1}}(p_{n+1})\,,
\end{aligned}
\tag{6}
$$

where the ν's are the helicities, the q's are the momenta of the gluons exchanged in the $\hat{t}$ channel, and $\hat{t}_i = q_i^2 \simeq -|q_{i\perp}|^2$. The Γ-tensors and the Lipatov vertex [4] are gauge invariant.
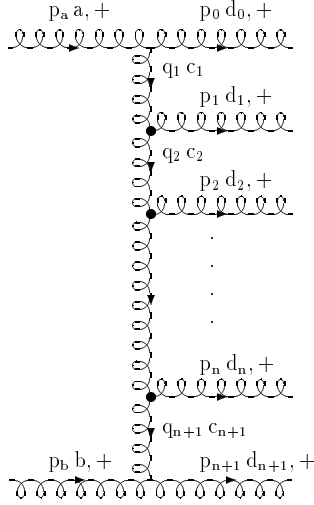

Figure 3: FKL amplitude for fixed gluon helicities. The blobs remind that Lipatov vertices are used for the gluon emissions along the ladder.

Helicity conservation at the production vertices for the first and the last gluon along

the ladder in eq.(6) yields the four helicity configurations (3). We choose then the configuration of Fig.3, which corresponds to the first of the configurations (3). There is however no restriction in eq.(6) on the helicities of the gluons produced from the Lipatov vertices along the ladder.

For consistency we use the representation of the gluon polarization used in the PT amplitudes [9],

$$\epsilon_\mu^\pm(p,k) = \pm \frac{\langle p \pm |\gamma_\mu| k\pm\rangle}{\sqrt{2}\langle k \mp |p\pm\rangle}, \tag{7}$$

where k is an arbitrary reference light-like momentum. For the polarization of gluons p_A and p_0 we choose p_B as reference vector, while for the polarization of gluons p_B and p_i with $i = 1, ..., n+1$ we choose p_A. The contraction of the Lipatov vertex with the gluon polarization (7) in eq.(6) is [10], [8],

$$\epsilon^+(p_i, p_A) \cdot C(q_i, q_{i+1}) = \sqrt{2} \frac{q_{i\perp}^* q_{i+1\perp}}{p_{i\perp}}. \tag{8}$$

The contractions of the helicity-conserving tensors with the gluon polarizations are [8],

$$\begin{aligned} \Gamma^{\mu_B\mu_{n+1}} \, \epsilon_{\mu_B}^{+*}(p_B, p_A) \, \epsilon_{\mu_{n+1}}^+(p_{n+1}, p_A) &= -\frac{p_{n+1\perp}^*}{p_{n+1\perp}}, \\ \Gamma^{\mu_A\mu_0} \, \epsilon_{\mu_A}^{+*}(p_A, p_B) \, \epsilon_{\mu_0}^+(p_0, p_B) &= -1. \end{aligned} \tag{9}$$

We rewrite then the product of structure constants in eq.(6) as the trace of a product of λ-matrices,

$$f^{ad_0c_1} f^{c_1d_1c_2} \cdots f^{c_nd_nc_{n+1}} f^{bd_{n+1}c_{n+1}} = -2\,(-i)^{n+2} \operatorname{tr}\left(\lambda^a \left[\lambda^{d_0}, \left[\lambda^{d_1}, ..., \left[\lambda^{d_{n+1}}, \lambda^b\right]\right]\right]\right), \tag{10}$$

which shows that also for the FKL amplitudes the only configurations which contribute are the 2^{n+2} color configurations which respect the rapidity ordering on the two-sided

plot. Substituting eq.(8), (9) and (10) into eq.(6), the FKL amplitude in the helicity configuration of Fig.3 is in agreement with eq.(5), thereby proving that the PT amplitudes and the FKL amplitudes coincide in the high-energy limit.

Finally, as remarked in the introductive paragraphs, including the leading logarithmic contributions, in $\ln(\hat{s}/\hat{t})$, of the loop corrections to eq.(6) modifies the propagator of the gluon of momentum q_i exchanged in the $\hat{t}$ channel only by a function of $\hat{t}_i$ [4]. Thus the FKL amplitude with the leading-logarithmic loop corrections retains the ladder structure of eq.(6) and the color structure of eq.(10), and the leading color configurations are still the ones whose untwisted lines are ordered in rapidity on the two-sided plot. Even though this is a simple observation from the standpoint of the FKL amplitudes, it is far from being obvious when we consider the color decomposition of multigluon amplitudes in a helicity basis, since the color structure of the tree-level amplitudes (2) does not describe all the possible color configurations of n gluons, more configurations appearing in the color decomposition at the loop level [11].

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